Quiz 10 11.8-and first part of 11.9/10

(1) Find the radius and interval of convergence.

(a) $\sum_{n=1}^{\infty} \frac{x^{2n}}{(2n)!}$ did in class

(2 points)

$$L=\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{x^{2(n+1)}}{(2(n+1))!}\cdot\frac{(2n)!}{x^{2n}}\right| = \lim_{n\to\infty}\left|\frac{x^{2n+2}}{(2n+2)!}\cdot\frac{(2n)!}{x^{2n}}\right|$$

$$\lim_{n\to\infty}\left|\frac{\cancel{x^{2n+2}}}{(2n+2)(2n+1)\cancel{(2n)!}}\cdot\frac{\cancel{(2n)!}}{\cancel{x^{2n}}}\right| = \lim_{n\to\infty}\left|\frac{x^2}{(2n+2)(2n+1)}\right| = 0 \text{ for all } x$$

$\boxed{R=\infty, (-\infty,\infty)}$

(b) $\sum_{n=1}^{\infty} \frac{8^n}{n}(x-2)^n$

(4 points)

$$L=\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{8^{n+1}(x-2)^{n+1}}{n+1}\cdot\frac{n}{8^n(x-2)^n}\right| = \lim_{n\to\infty} 8\left(\frac{n}{n+1}\right)|x-2| = 8|x-2|$$

$\frac{n}{n+1}\to 1$

If $L<1$, series is A.C. $8|x-2|<1 \Rightarrow |x-2|<\frac{1}{8}=R$

$-\frac{1}{8}<x-2<1/8$

$\frac{15}{8}<x<\frac{17}{8}$

check endpoints

$L=1$

$x=\frac{15}{8}$: $\sum_{n=1}^{\infty}\frac{8^n}{n}(x-2)^n = \sum\frac{8^n}{n}\left(-\frac{1}{8}\right)^n = \sum\frac{(-1)^n}{n}$ conv

$x=\frac{17}{8}$: $\sum\frac{8^n}{n}\left(\frac{1}{8}\right)^n = \sum\frac{1}{n}$ DIV

$\boxed{\left[\frac{15}{8}, \frac{17}{8}\right)}$

(c) $\sum_{n=1}^{\infty} n!(x+4)^n$

(2 points)

$$L=\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)!(x+4)^{n+1}}{n!(x+4)^n}\right| = \lim_{n\to\infty}|n+1|(x+4) = \infty$$

for all x except $x=-4$. When $x=-4$, series is $\sum_{n=1}^{\infty} n!(0)^n = 0$, so converges at $x=-4$ only

$\boxed{R=0}$

(2) Use the definition of a Taylor series to find the first four nonzero terms of the series for $f(x) = \tan x$ centered at $a = \frac{\pi}{4}$ Directly :

(5 points)

$$f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 +$$

	at x	at x=a= $\frac{\pi}{4}$
f	$\tan x$	1
f '	$\sec^2 x$	2
f ''	$2\sec^2 x \tan x$	4
f '''	$4\sec^2 x \tan^2 + 2\sec^4 x$	16
f ^{n}	f ^{n}(x)=	f ^{n}(0)=

$$\overset{1}{f(a)} + \overset{2}{f'(a)}(x-a) + \frac{\overset{4}{f''(a)}}{2!}(x-a)^2 + \frac{\overset{16}{f'''(a)}}{3!}(x-a)^3 +$$

$$\boxed{1 + 2\left(x-\frac{\pi}{4}\right) + 2\left(x-\frac{\pi}{4}\right)^2 + \frac{8}{3}\left(x-\frac{\pi}{4}\right)^3}$$

(7 points)

2) Find the Taylor series for $f(x) = \cos x$ centered at $a = \frac{\pi}{2}$ using the definition. Express in summation

(7 points)

	at x	at x=a= $\pi/2$
f	$\cos x$	0
f '	$-\sin x$	-1
f ''	$-\cos x$	0
f '''	$\sin x$	1
	$\cos x$	0
f ^{n}	f ^{n}(x)=	f ^{n}(0)=

$$\cos x = -\left(x-\frac{\pi}{2}\right) + \frac{1}{3!}\left(x-\frac{\pi}{2}\right)^3 + \frac{1}{5!}\left(x-\frac{\pi}{2}\right)^5 +$$

$$= \boxed{\sum_{n=0}^{\infty} \frac{(-1)^{n+1}\left(x-\frac{\pi}{2}\right)^{2n+1}}{(2n+1)!}}$$

$$f(x) = \overset{0}{f(a)} + \overset{-1}{f'(a)}(x-a) + \frac{\overset{0}{f''(a)}}{2!}(x-a)^2 + \frac{\overset{1}{f'''(a)}}{3!}(x-a)^3 + \frac{\overset{0}{f^{(4)}(a)}}{4!}(x-a)^4 + \cdots = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$